Teacher notes
Topic B

The equilibrium temperature of the earth

In the textbook we derived that the equilibrium temperature of the earth is given by

$$T=\left(\frac{1-\alpha}{\sigma}\frac{S}{4}\right)^{\frac{1}{4}}$$

where α is the albedo of the planet and S is the solar constant (the intensity of the solar radiation at the upper atmosphere). The factor of 4 was explained by the fact that the radiation incident on earth goes through an area πR^2 but is averaged over the entire earth surface area which is $4\pi R^2$. Here is a slightly alternative derivation.

The intensity of the solar radiation at the upper atmosphere is S. Of this, a fraction α is reflected so a fraction $(1-\alpha)S$ is absorbed. Notice that the Sun radiates mainly in the UV and so the albedo α is the albedo appropriate to UV wavelengths. Thus, the *power* absorbed by the surface is

$$P_{\text{abs}}=IA=\pi R^2(1-\alpha)S$$

because the earth projects an area πR^2 to the incident radiation.

The *entire* surface radiates and so the *power* radiated is

$$P_{\text{rad}}=\varepsilon\sigma 4\pi R^2T^4$$

where ε is the emissivity of the surface. The surface radiates mainly in the IR and so ε is the emissivity appropriate to IR wavelengths. For equilibrium we have

$$\varepsilon\sigma 4\pi R^2T^4=\pi R^2(1-\alpha)S$$

which leads to

$$T=\left(\frac{(1-\alpha)}{\varepsilon\sigma}\frac{S}{4}\right)^{\frac{1}{4}}$$

So, the factor of 4 appears here in a very slightly different way that students may find easier to understand.

It is customary to take $\varepsilon = 1$, i.e. we assume a black body surface (but not a black body atmosphere). In that case we get the standard formula $T = \left(\frac{1-\alpha}{\sigma}\frac{S}{4}\right)^{\frac{1}{4}}$. This gives the standard result T = 254 K for S = 1360 W m^{-2} and α = 0.31.

Notice however, that since $\varepsilon < 1$ (the surface is not a perfect black body), the equilibrium temperature would be *a bit higher* than 254 K.

Some students argue that we should write $1-\alpha = \varepsilon$ (because they have seen this formula in various places) in which case the expression $T = \left(\frac{(1-\alpha)}{\varepsilon\sigma}\frac{S}{4}\right)^{\frac{1}{4}}$ becomes $T = \left(\frac{S}{4\sigma}\right)^{\frac{1}{4}}$ and evaluates to 278 K independently of the albedo value! This gives a value close to the actual average temperature of the earth without making use of the greenhouse effect!

The argument for $1-\alpha = \varepsilon$ is simply this: an intensity I is incident on a surface. A fraction αI is reflected and a fraction εI is absorbed. Since *no energy gets transmitted anywhere* conservation of energy demands $\alpha I + \varepsilon I = I$ hence $1-\alpha = \varepsilon$.

But this argument is wrong!

The reason has already been hinted at above. Albedo and emissivity are wavelength dependent. In our case the albedo is an UV albedo and the emissivity is an IR emissivity. *At the same wavelength* we would have $1-\alpha = \varepsilon$ but not so otherwise.